Machine learning of interatomic interaction

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Skoltech, AMM course 19 May 2020 www.aflowlib.org/superalloys/



Prediction of convex hull of stable alloys



Machine-learning interatomic potentials

My perspective

Machine learning as interpolation,

... data-driven and multidimensional.

- Problem: Given E^{qm}(X), interpolate
 it with E(X)
- Issue: no transferability w.r.t. the number of atoms
- Solution: <u>use locality</u>! (An atom interacts only with 10-100 neighboring atoms)



Traditional fitting

- Embedded atom model: $E = \sum_{i} V(r_{i1}, r_{i2}, ...)$,
- $V(\mathbf{r}_i) = \sum_j \varphi(r_{ij}) + F(\sum_j \rho(r_{ij})).$
- Early interatomic potentials (=force fields) had few (three) parameters fitted from few experimental data (elastic constants, defect formation energy, etc.)
- Later potentials have tens of coefficients (e.g., spline coefficients) fitted from the QM data.
- What is different now: there are lots of data!
- So, the question is: *how to incorporate lots of data into the models*?



Machine-learning ideology:

- 1. Choose a (machine-learning) model E = E(x)(x is an atomic configuration)
- 2. We want to minimize $|E^{qm} E|$. So we:
- Generate data: $x^{(1)}$, $x^{(2)}$, ...; $E^{qm}(x^{(1)})$, $E^{qm}(x^{(2)})$,..., $f^{qm}(x^{(1)})$, ...
- Minimize on data: $\sum_i |E(x^{(i)}) E^{qm}(x^{(i)})|^2 + (\text{forces}) + ...$

But what if sampling the right $x^{(i)}$ is a part of the problem?

Illustration: calculating convex hull



Problem:

accurate sampling of ground state structures

needs

- accurate approximation of PES which needs
- accurate sampling of ground state structures

which needs ...

Solution: Active learning / Learning on-the-fly

Active learning simulation **Molecular Simulation** energy, forces molecule Active learning Machine needs yes learning no learning? potential (MLP) Get QM data

Overview

- **1.** Overview
 - Why is this important?
- 2. Moment Tensor Potentials
- 3. Active learning (how to learn while sampling a PES)
- 4. Applications

A dream of comp.mater.sci



Molecular modeling

• ~40% of supercomputing time is spent on Molecular Modeling



[Adopted from nersc.gov]

Molecular dynamics scales



Overview

1. Overview

2. Machine-learning Potentials

- 3. Active learning (how to learn while sampling a PES)
- 4. Applications

Regression of Atomistic Properties

Basic problem:

- Given a molecule/atomic system r_i predict its property $F(r_i)$.
- Often, one does want to learn the physical symmetries (or, more generally physical properties), instead embed into the model.
- For interatomic potentials Step 1 is to use locality

Locality: Energy

$$E = \sum_{i} V(r_{i1}, r_{i2}, ...)$$

- Most interatomic potentials are covered. (Coulomb should be added explicitly.)
- Problem: find a good V.
- This step gives us:
 - Transferability wrt number of atoms (can apply to systems with millions of atoms)
 - translation invariance



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 - translation invariance
- To do next: rotation and permutation invariance



Rotation and permutation invariance:

By far most popular solution is to design descriptors of atomic environments

• Behler-Parrinello descriptors:

$$D^{(2)}(\boldsymbol{r}_{i}) = \sum_{j} f(r_{ij}) \text{ for some scalar function } f$$

$$D^{(3)}(\boldsymbol{r}_{i}) = \sum_{j} \sum_{k} f(r_{ij}) f(r_{ik}) \varphi(r_{ij} \cdot r_{ik}) \text{ for some scalar } f \text{ and } \varphi$$

For long time it was considered that they could generate a complete description of atomic environment, but recently it was proved that this is false: <u>https://arxiv.org/pdf/2001.11696.pdf</u>

Regression: Neural networks

- Problem: given a vector of descriptors v_1, \dots, v_M , find the mapping $F = F(v_1, \dots, v_M)$
- Machine-learning approach: find $F = F(v_1, ..., v_M)$ from data by fitting some parameters
- Two-level Neural network:

$$F(\boldsymbol{v}) = A_2 f(A_1 \, \boldsymbol{v} + \boldsymbol{b}_1) + \boldsymbol{b}_2,$$

Where matrices A_1, A_2 and vectors $\boldsymbol{b}_1, \boldsymbol{b}_2$ are found from data

Alternative: Gaussian process regression

- $F(\mathbf{r}_{i}) = \sum_{l} k(\mathbf{r}_{i}, \mathbf{r}_{i}^{(l)})$, where k is a kernel giving a similarity measure between the given atomic environment \mathbf{r}_{i} and those from the training(=fitting) set $\mathbf{r}_{i}^{(l)}$.
- The problem reduces to designing a kernel satisfying physical symmetries

Alternative: Moment Tensor Potentials

Descriptors of atomic environments:

- Moments of inertia of surrounding atoms
- They satisfy the needed symmetries (rotation, permutation, translation, ...);
- Math:





Moment Tensor Potentials, basis functions

- $V(\boldsymbol{u};\theta) = \sum_{\alpha} \theta_{\alpha} B_{\alpha}(\boldsymbol{u})$
- $B_{\alpha}(\mathbf{u})$ are (all) different multiplications (contractions) of inertia tensors $M_{m,n}(\mathbf{u})$ yielding a scalar.

Theorem:

• $B_{\alpha}(\mathbf{u})$ is a complete basis

Learning curves

Database (Csanyi, Bartok, Szlachta, 2014)

• Tungsten: uniform and perturbed lattices, vacancies, dislocations



Performance tests

Database (Csanyi, Bartok, Szlachta, 2014)

• Tungsten: uniform and perturbed lattices, vacancies, dislocations

Potential:	GAP	MTP_1	MTP_2
CPU time/atom [ms]:	134	2.9	0.8
basis functions:	10000	11133	760
Fit errors:			
force RMS error [eV/Å]:	0.0633	0.0427	0.0633
[%]:	4.2%	2.8%	4.2%
Cross-validation errors:			
force RMS error[eV/Å]:	-	0.0511	0.0642
[%]:	-	3.4%	4.3%

Comparison with more methods



Yunxing Zuo, Chi Chen, Xiangguo Li, Zhi Deng, Yiming Chen, Jörg Behler, Gábor Csányi, A.S., Aidan P. Thompson, Mitchell A. Wood, Shyue Ping Ong. arXiv:1906.08888

Training and Validation

Often, to test the quality of the potential we split training and validation:

- Machine-learning model: $E = E(\theta, x)$
- Training set: $\boldsymbol{x}_{tr}^{(1)}$, ..., $\boldsymbol{x}_{tr}^{(N)}$. Training: $\min_{\boldsymbol{\theta}} \sum_{k} \left(E\left(\boldsymbol{\theta}, \boldsymbol{x}_{tr}^{(k)}\right) E_{tr}^{(k)} \right)^2$
- Validation set: $x_{vld}^{(1)}$, ..., $x_{vld}^{(N)}$. Validation error:

$$\left(\frac{1}{K}\sum_{k}\left(E\left(\boldsymbol{\theta},\boldsymbol{x}_{\text{vld}}^{(k)}\right)-E_{\text{vld}}^{(k)}\right)^{2}\right)^{\frac{1}{2}}$$

Training and Validation



Active Learning of Interatomic Potentials



Active Learning of MLIP: Motivation

Higher accuracy => More parameters to fit => Lower transferability


















































Active learning: it's about reliability, not accuracy

- Fitting $E(x) = x^2 + x^3 \exp(-x^2/2)$ with $E(x) = c_1 x^2 + x_2 x^3$
- Red: Minimizing the error on an "exact, infinitely long" MD, error=0.25
- Blue: Active learning, error=0.46



How we do it?

D-optimality

Skip to Applications

D-optimality

essentially

- detects hitting outside a convex hull,
- but for linear models

(convex hull -> simplex)



E. Podryabinkin, A. Shapeev (2017)

Active Learning (AL) of Interatomic Potentials

• Fitting equations (overdetermined):

• Its matrix:
$$B = \begin{pmatrix} b_1(\mathbf{cfg}^{(\mathbf{k})}) & \dots & b_N(\mathbf{cfg}^{(\mathbf{k})}) \\ \vdots & \ddots & \vdots \\ b_1(\mathbf{cfg}^{(K)}) & \dots & b_N(\mathbf{cfg}^{(K)}) \end{pmatrix}$$

- D-optimality criterion: find an $N \times N$ submatrix A with largest |det(A)|
- Selecting rows = selecting configurations

AL in practice

MAXVOL algorithm (*Goreinov et al., 2010*):

• Given:

• Current set
$$A = \begin{pmatrix} b_1(\mathbf{cfg}^{(1)}) & \dots & b_N(\mathbf{cfg}^{(1)}) \\ \vdots & \ddots & \vdots \\ b_1(\mathbf{cfg}^{(N)}) & \dots & b_N(\mathbf{cfg}^{(N)}) \end{pmatrix}$$

- Candidate (new) cfg*
- Define extrapolation grade (EG) = factor by which $|\det A|$ can increase
- EG<1 = interpolation. EG > 1 + ϵ = include **cfg**^{*} in the training set
- Can be done at $O(N^2)$ complexity

AL: interpretations

Geometric interpretation

- The vector $b_1(\mathbf{cfg}^*)$... $b_N(\mathbf{cfg}^*)$ is a descriptor in an N-dimensional space
- Increasing | det A | = increasing the volume of the simplex based on cfg⁽¹⁾, ..., cfg^(N)

Information-theoretic interpretation

• information = log |det A|. Configuration is trained on if this increases the information.

AL: interpretations

Statistical interpretation

• If $E^{qm,(1)}$, ..., $E^{qm,(N)}$ have random independent noise, then the noise in the model is minimized

Algebraic interpretation

• It can be shown that $E(\mathbf{cfg}^*) = \sum_i c_i E^{qm,(i)}$, hence all $|c_i| \le 1 \Leftrightarrow E(\mathbf{cfg}^*)$ is interpolated through $E^{qm,(i)}$.

Applications

Application #1: Learning on the fly



- Combines training and evaluation of MLIP
- Detects and learns "extrapolative" configurations
- Robust
- Balancing accuracy and amount of QM calcs

Application example #0: Learning on the fly in MD process at NVT-ensemble of 128 BCC-Li atoms



Conclusion: Amount of QM calcs can be reduced several times at the cost of minor losses in accuracy

www.aflowlib.org/superalloys/



Prediction of convex hull of stable alloys

How it is done:

- 1. Start with 1500 crystal prototypes (unequilibrated structures)
- 2. Equilibrate (relax) them with DFT and choose the ones on the convex hull



Convex hulls now

How it is done:

- 1. Start with 400K crystal prototypes (unequilibrated structures)
- 2. Equilibrate (relax) them with MLIP while learning on the fly

K. Gubaev, E. Podryabinkin, Gus L.W. Hart, A.S. (2019)



Results

• Some newly discovered structures are hard to "sample passively":



Convex hulls now: details

1. Screen-1:

- 1. Start with **400K** structures
- 2. Obtain **400K** relaxed structures, with RMSE = **25** meV/atom
- 3. Retain **60K** low-energy structures (within $4-\sigma$)
- 2. Screen-2:
 - 1. Start with **60K** structures
 - 2. Obtain **60K** relaxed structures, with RMSE = **8** meV/atom
 - 3. Retain **7K** low-energy structures (within 4-σ)

ΑI

- 3. Final relaxation:
 - 1. Relax **7K** structures on DFT



Results and Discussion

- No approximation error in the answer! (We only take a risk of missing a structure in the 4- σ interval.)
- 100x speed-up; CPU time:
 - 1. Final relaxation: 90%
 - 2. Training set: 9%
 - 3. Training, Relaxation: 1%
- Main challenge: reduce the 90% \Leftarrow improve accuracy (8 meV/atom):
 - Go beyond local environments (we quickly reach the limit with local interaction)
 - Include spins (suffer from "jumping" from nonmagnetic PES to ferromagnetic PES)
 - Periodic table-wide potential (reuse data from old systems from new systems)
 - Better uncertainty estimation (better than just 4- σ).
- Sampling is now the bottleneck, not DFT (we should make friends with Complex High-Dimensional Energy Landscapes)

On-lattice models: Cluster expansion

- Atoms of different kind sit in the lattice sites.
- Problem: predict the interatomic interaction energy (formation energy, mixing enthalpy)

E(
$$V_1(\bullet) + V_1(\bullet) + ...$$

+ $V_2(\bullet \bullet) + V_2...$
+ $V_3....$



On-lattice models: Potentials

- Atoms of different kind sit in the lattice sites.
- Problem: predict the interatomic interaction energy (formation energy, mixing enthalpy)




Comparison with existing methods: without local lattice distortions



[2] Fritz Körmann, Andrei V Ruban, and Marcel HF Sluiter. Long-ranged interactions in bcc NbMoTaW high-entropy alloys. Materials Research Letters, 5(1):35-40, 2017.

Results & discussion: accounting for local lattice distortions



"Perfect crime" of machine-learning potentials

100-fold speed-up with no detectable trace of using machine learning in the final result (in short, a free lunch)

<u>Alexander Shapeev</u>¹, Konstantin Gubaev¹, Evgeny Podryabinkin¹, Gus Hart²

1: Skoltech (Moscow, Russia)

2: BYU (Provo, Utah)

2019 APS Meeting, Boston 04 March, 2019

Application 1b: Boron crystal structure prediction

E. Podryabinkin, E. Tikhonov, A.S., Artem Oganov (2019)

Boron structures prediction challenges:

- A lot of allotropes
- Some allotropes has more than 100 atoms (impossible with DFT)
- Small energy/atom difference between structures with PES minima





B-28 (6.678eV/atom)2 months with DFT5 days with MLIP



B-54 (6.667eV/atom) 2 year on DFT 8 days on MLIP



B-106 Best Found on MLIP within 2 weeks



B-108 = B12 x9 10 years with DFT 2 weeks with MLIP

Application #2: thermodynamic integration

• Vibration entropy of a MoNbTaVW quasi-random structure

Blazej Grabowski, Yuji Ikeda, Fritz Koermann, Christoph Freysoldt, Andrew Duff, A.S., Joerg Neugebauer (2019)



Application #3: elastic properties

A.S., E. Podryabinkin, K. Gubaev, F. Tasnadi, Igor Abrikosov (manuscript)

• Elastic constants $C_{11} > C_{12} > C_{44}$ (bcc-Ti)



DFT with uncertainty (50000 DFT-MD time steps)

MTP (negligible statistical uncertainty)

We trade

- 1 GPa statistical error for
- 1 Gpa model error and
- 1000x speed-up

Application #4: Calculation of Diffusivities

Ivan Novoselov, E. Podryabinkin, A.S., Alexey Yanilkin (2019)

Potential advantages:

- MTP: accurate description of lowsymmetry configurations (e.g. saddle point)
- Active learning: rapid exploration of phase space
- Learning on the fly: effective sampling of rare events



Application #5: Molecular reaction rates

I. Novikov,

Y. Suleimanov, A.S. (2018)

• Use RPMD + MTP

•	Rate (<i>cm</i> ³ /s)	Existing PES (reference)	AL-MLIP
•	Classical	4.5×10^{-14}	4.1×10^{-14} (9% error)
	Quantum-corrected (128 RPMD-beads)	2.5×10^{-12}	2.1×10 ⁻¹² (20% error)



Application #6: automated phase diagrams

Conrad Rosenbrock, Livia Bartok-Partay, Noam Bernstein, K. Gubaev, Gabor Csanyi, A.S., Gus Hart (manuscript)

 Fitted a potential for Ag-Pd binary system (solid and liquid)

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On-lattice models: HEAs

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T. Kostiuchenko, Fritz Koermann, Joerg Neugebauer, A.S. (2019)



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Results & discussion: accounting for local lattice distortions



Summary: MLIP Code

- Public version: <u>http://mlip.skoltech.ru/</u>
 - developer's version (incl. unpublished capabilities) by request
- QM model interfaces:
 - VASP, Gaussian (DFT)
 - PROFESS (OFDFT)
- Atomistic Driver interfaces:
 - LAMMPS, serial and parallel (but no learning on the fly)
 - USPEX
 - ASE
 - RPMDrate
- Active learning / Learning on the fly

Related fields

- Learning Potential Energy Surfaces (PES) of molecules
 - Similar idea, but no locality. Can be described, e.g., by all N (N+1)/2 pairwise distances in the system
 - Older field (started before 2000),
 - first time neural networks were applied to chemistry
- Cheminformatics
 - Structure-property relations learning things other than energy
 - next lecture