

## Advanced Materials Modeling Homework 2

**Notes:** In multiple choice problems explain your answer. Add references if needed. Upload solution as a single file “YourName.pdf” or “YourName.zip”.

1. Derive the dependence of total energy  $E_{\text{tot}}(N)$  of a system on the number of electrons between two integer values.

Hint: Minimize the total energy

$$E_{\text{tot}} = p_N E_N + p_{N+1} E_{N+1} + p_{N-1} E_{N-1},$$

where  $p_i$  is the probability of charge state  $i$ , with respect to  $p_i$  under the following constraints: condition of convex dependence on *integer*  $N$ :

$$E_{N+1} + E_{N-1} > 2E_N;$$

the normalization condition

$$p_N + p_{N+1} + p_{N-1} = 1;$$

and the fixed charge constraint

$$p_N N + p_{N+1} (N + 1) + p_{N-1} (N - 1) = N + \omega,$$

where  $\omega$  is the fractional charge.

2. Prove Janak’s theorem:

$$\frac{\partial E[f, n]}{\partial f_i} = \varepsilon_i,$$

where  $E[f, n]$  is the total energy depending on occupations  $f$  and electron density  $n(\mathbf{r})$ , and  $\varepsilon_i$  is the energy of the  $i$ th Kohn-Sham eigenstate.

Hint:

$$E[f, n] = -\frac{1}{2} \sum_i f_i \langle \psi_i | \nabla^2 | \psi_i \rangle + \int d^3 r V_{\text{ext}}(\mathbf{r}) n(\mathbf{r}) + \frac{1}{2} \int d^3 r d^3 r' \frac{n(\mathbf{r}) n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + \int d^3 r V_{\text{XC}}(\mathbf{r}) n(\mathbf{r}),$$

$$n(\mathbf{r}) = \sum_i f_i |\psi_i(\mathbf{r})|^2,$$

$$\int d^3 r |\psi_i(\mathbf{r})|^2 = 1,$$

and Kohn-Sham equations:

$$-\frac{1}{2} \nabla^2 \psi_i(\mathbf{r}) + V_{\text{ext}}(\mathbf{r}) \psi_i(\mathbf{r}) + \left( \int d^3 r' \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \right) \psi_i(\mathbf{r}) + V_{\text{XC}}(\mathbf{r}) \psi_i(\mathbf{r}) = \varepsilon_i \psi_i(\mathbf{r}).$$

Also, consider  $\psi_i$  and  $V_{\text{XC}}$  as independent of  $f_i$  (PARTIAL derivative with respect to  $f_i$ !).

**3.** Mark all correct statements:

Hartree-Fock method

- (A) includes correlation
- (B) is exact
- (C) is self-interaction free
- (D) generally underestimates band gaps

**4.** Mark all correct statements:

DFT with LDA functional

- (A) includes correlation
- (B) prefers electron delocalization
- (C) is self-interaction free
- (D) generally underestimates band gaps

**5.** Mark all correct statements:

For exact DFT functional

- (A) KS gap is equal to observable gap
- (B) highest occupied Kohn-Sham state energy does not depend on occupation  $0 < f < 1$
- (C) the total energy of a system is a non-linear function of particle number between integer occupations